1. **Cosmic-ray flux, the knee and event rates**
   One of the most distinct features in the cosmic-ray spectrum is the change of the power-law around the knee energy. The differential intensity below and above the knee energy ($\approx 3*10^{15}$ eV) can be expressed as:

$$\mathrm{d}j/\mathrm{d}E = C_1 * E^{-2.7}; E < 3*10^{15}\text{eV}$$

$$\mathrm{d}j/\mathrm{d}E = C_2 * E^{-3.1}; E \geq 3*10^{15}\text{eV},$$

   where C_1 and C_2 are constants.

   Given that the integrated intensity above $E = 10^{15}$ eV is $J(> 10^{15}\text{eV}) = 1.6*10^{-10}$ cm^{-2} sr^{-1} s^{-1}:

   a) Work out a formula for the integrated intensity above an energy E_0 ($J(E > E_0)$). Hint: Write the integral $J = \int_{E_0}^{\infty} \mathrm{d}j/\mathrm{d}E \, \mathrm{d}E$ and use the requirement that the spectrum is continuous also at the knee energy in order to obtain C_1 and C_2.

   b) AMS02-spectrometre measures the spectrum of cosmic rays. Estimate the number of events the spectrometre will record in one year, above the energies of 10^{14} eV and 10^{15} eV.

   c) Kascade - EAS-array measured the cosmic-ray composition changes around the knee energy. The array consisted of 252 detector stations arranged on a rectangular grid with a distance of 13 meters to each other. For proper analysis of the air shower it is required that the central axis of the shower is located within the array area. Estimate the number of such air showers recorded per year by the array.

   d) The cosmic-ray spectrum exhibits a change in slope also at the energy of $E \approx 10^{18}$ eV ("the ankle"). In order to investigate the spectrum at this energy, how large an air-shower array should be?

   Hint: In general the number of events recorded above an energy E_0 is $N = \int T * A(\theta, \phi) * J(E_0) * d\Omega$, where T is the duration of measurement, $A(\theta, \phi)$ the projected detection area perpendicular to a given shower direction (θ, ϕ) and the integral is over the full solid angle. You may use simple estimates for the angular coverage (acceptance) of the different experimental setups and assume A independent of angles.

2. **Extensive air shower, simplified theoretical investigations**
a) A very simple shower model of EAS is the so-called 'Heitler model'. It assumes that the incident particle of energy E_0 splits in to 2 particles of equal energy after propagating a distance λ. The splitting process continues as the 2 particles of energy $E_0/2$ split in to 4 particles of equal energy, etc. The splitting process stops once the particle energies are below a critical energy E_c, after which most of the energy is lost through ionization processes and particle numbers decrease. Work out a formula for the maximum number of particles in the shower and the depth of the shower maximum as a function of E_0 and E_c.
b) Assuming that the first interaction occured at the top of the atmosphere ($\lambda_I \approx 50$ g/cm^2), $E_c = 84$ MeV and $\lambda = 37$ g/cm^2 calculate the maximum number of particles and the depth of the shower maximum of the EAS, if the initial energy was $E_0 = 10^{15}$ eV. What if the energy was $E_0 = 10^{20}$ eV? To which altitude (in kilometres) do the maximum depths correspond to?
c) In the superposition model the interaction of nucleus of the total energy of E_0 and which consists of A nucleons can be described as A interactions of one nucleon of energy E_0/A. Workout the formulae for the maximum number of particles and the depth of shower maximum as a function of A, E_0 and E_c. Compare the formulae obtained with that of a) and conclude about a one main difference of proton and heavy nuclei initiated air showers.
d) The muon component of the EAS arises mainly due to the decay of charged pions $\pi^+ \to \mu^+ + \overline{\nu}_\mu$. The interaction lenght of charged pions is $\lambda \approx 120$ g/cm^2 and the mean life of charged pion is $\tau_{\pi^+} = 2.6 * 10^{-8}$ s. Consider a 20 GeV pion at the altitude of 10 kilometres. Is it more likely that the pion decays or interacts with the air nuclei? Mass of the pion is 140 MeV/c^2.
The results of a), b) and c) follow from simplifications. You may want to also compare the resulting values with that calculable from the formulae given in the lecture notes [slide 3.21].

3. **GZK-cut**
Very high-energy cosmic-ray protons may loose their energy as they interact with cosmic microwave background photons:

$$p + \gamma_{\text{CMB}} \to p + \pi^0$$

The temperature of CMB in the "laboratory system" (as seen in Earth) is $T \sim 2.7$ K.
a) Calculate the minimum energy of the proton, for which the above process is possible. You may assume that the energy of the photon is $E_\gamma = k_B T$, where K_B is the Bolzmann constant.
Hints: Examine the reaction in centre of mass coordinates ($\overline{p}_p = -\overline{p}_\gamma$). What is the required photon energy E_γ^{CM} for a final state of proton and a pion in rest? ($m_p = 938.3$ MeV/c^2, $m_{\pi^0} = 135.0$ MeV/c^2.)
Make a Lorentz-transformation to a "laboratory frame", in a way that the energy of the photon corresponds to the CMB energy. Apply the transformation to the proton.

b) How high (on earth) will a one kilogram weight be lifted with the energy?

Compare the calculated proton energy to that given in the lectures [slide 3.13]. Why does the calculated answer differ from the given?

4. **Extended air showers, considerations about measurement**
a) The figure below depicts a shower longitudinal profile (depth vs. number of particles) recorded by the Fly's Eye experiment during the year 1993. What can you say about the properties (energy and mass) of the cosmic ray?
b) Why are the EAS detectors very often located at high altitudes (on mountain regions)? Give at least two physics arguments.