1. **Acceptance, Intensity, Flux**

A) The event rate is

$$N/T = \int_\Omega I(\theta,\phi) * A(\theta,\phi) d\Omega.$$

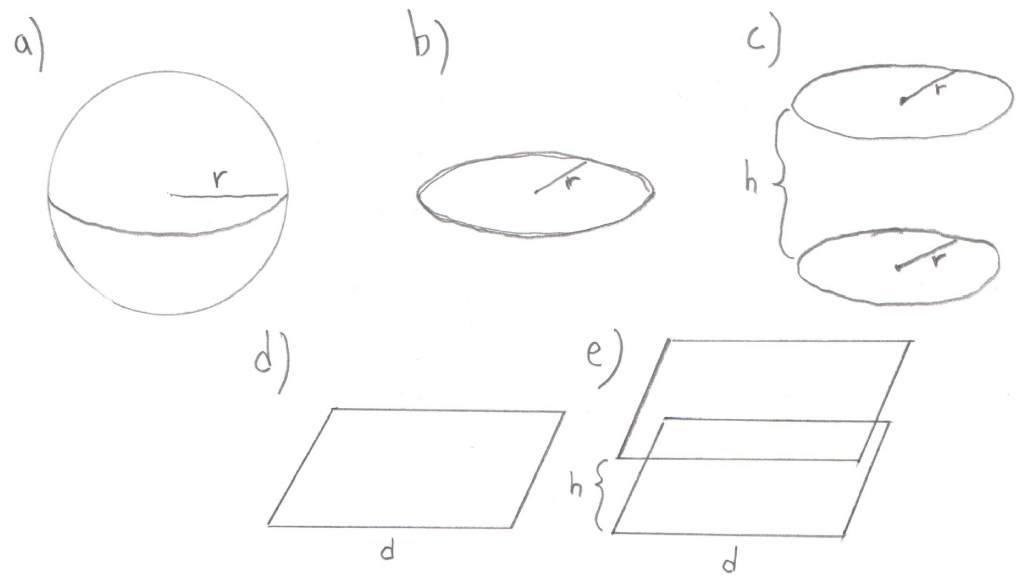
Let us now define the geometrical acceptance G of a detector as

$$G = \int_\Omega A(\theta,\phi) d\Omega.$$

Here N is number of events, T is time, $I(\theta,\phi)$ is intensity, $A(\theta,\phi)$ is the projected detector area as viewed from the direction (θ,ϕ), $\theta = \{0,\pi\}$ is the zenith angle and $\phi = \{0, 2\pi\}$ is the atsimuth angle.

Let us consider five different detector geometries. Calculate or reason the geometrical acceptance of the following:
a) Spherical detector with radius r.
b) Circular detector plane of radius r (placed horizontally).
c) Two circular detector planes of radius r (placed horizontally) with a vertical separation of h. Here a particle, whose trajectory is a straight line, is detected if it traverses through both circular planes. You may use suitable approximations.
d) A square detector plane of side lenght of d (placed horizontally).
e) Two square planes of side lenght of d (placed horizontally) with vertical separation of h. Here a particle, whose trajectory is a straight line, is detected if it traverses through both square planes.
[**c) and e) are a bit harder, you may calculate only a), b) and d). You get an extra point for calculating e) correctly! You may use a computer.**]

B) Consider an *isotropic* flow of particles with intensity $I = 1\ \text{m}^{-2}\text{s}^{-1}\text{sr}^{-1}$. What is the particle counting rate of each of the apparatus a)-e)? Use $r = 1$ m, $d = 1$ m and $h = 3$ m.
C) Assume a directed uniform flux of particles originating from a far-faraway source (flux density at the detector $F = 1\ \text{m}^{-2}\text{s}^{-1}$). What is the particle counting rate of each of the apparatus a)-e) if the source is in the direction directly above the apparatus? What if the source is to the left?
D) Assume that the Kascade-array records only the events for which $\theta < 35$ degrees. How many events does the array record per year between the energy interval $10^{15} - 10^{16}$ eV? Hint: repeat Exercise 2.1 c) by taking into account the results and formulas from A). Now the formula will be $N = \int_\Omega T J(E_0) A(\theta,\phi) d\Omega \rightarrow 2\pi \int_0^{35deg} J(E_0) A \cos(\theta)\sin(\theta) d\theta$, where A is the area of the array.

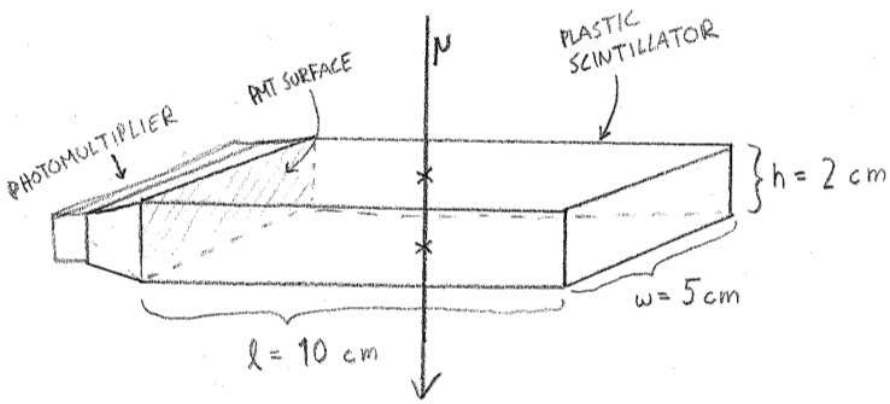


Schematic representations of the apparatus of task 1

2. **Detection techniques**

a) Cherenkov radiation is produced if a charged particle exceeds the light speed in the material. What is the minimum energy of an electron to cause Cherenkov radiation in water (use the refractive index $n = 1.33$)? What is the minimum energy of a muon? Derive the formula for the angle of the Cherenkov cone [lecture slide 8.19].

b) Consider the following geometry of a plastic scintillator and a photomultiplier:



Assume one vertical muon traversing through the middle of the scintillator. The energy loss of a muon in a plastic scintillator detector is $E \approx 2$ MeV/cm [lecture slides 8.1-8.7]. For simplicity, assume that all the energy loss in the scintillator material is converted to monoenergetic photons of the wavelenght of $\lambda \approx 450$ nm. Estimate the number of photons arriving to the photomultiplier surface.

In more detail, the light yield of a scintillator $\epsilon_{\rm sc}$ describes the fraction of energy loss going into photons, and the scintillator material is not transparent, but light is attenuated in it. If the light yield $\epsilon_{\rm sc} = 0.1$ and the light attenuation lenght $\lambda = 100$ cm, how many photons arrive to the photomultiplier surface?

How could one modify the detector in order to increase the number of photons arriving to a photomultiplier surface?

c) Two plastic scintillators of the size 0.1 m^2 are placed directly on top of each other. Assume that both scintillators measure the natural gamma-ray background with a singles rate of 100 Hz. In the measurement a coincidence is counted if a signal from both scintillators is recorded within a time window of 100 ns. Estimate the rate of accidental coincidences due to the natural gamma-ray background. Compare the rate with the muon flux through the scintillator setup which we may take to be $10/s$. [see lecture slide 8.16]

d) Go through the lecture slides 8.1 - 8.22. What are the different particle detection techniques described? What are the similarities and the differences of them? Give one benefit of applying each technique for the measurement of particles.

3. **Radiochemical neutrino measurements, the Homestake Chlorine experiment**

The radiochemical Chlorine experiment in the Homestake Gold Mine (~4100 mwe) measured the (^{8}B and ^{7}Be) solar neutrino flux. The experiment consisted of one steel tank containing in total 615 tons of C_2Cl_4 and aimed to observe the reaction

$$\nu_e +^{37}\mathrm{Cl} \rightarrow^{37}\mathrm{Ar} + \mathrm{e}^- .$$

a) Calculate the threshold energy of the reaction. Hint: what is the minimum energy required for the neutrino?

b) Calculate the number of Cl atoms in the tank.

c) Assume that the neutrino flux density (over the threshold energy) is $\approx 2 * 10^6$ cm^{-2} s^{-1}. Calculate the rate of argon production in the tank. Calculate the number of argon atoms produced during one month of exposure. Use a cross section of $\sigma = 1.14 * 10^{-42}$ cm^2.

d) ^{37}Ar decays by electron capture reaction (half-life $t_{1/2} = 35.0$ days). After a one month of exposure to the neutrino flux, the argon atoms were chemically extracted from the tank and each of the argon-atom decays were observed and counted in a proportional counter (one by one). Calculate the number of argon-decay reactions observed in the proportional counter.

Hints for d): assume that the tank contains no argon atoms at the beginning of the one month exposure. Take into account the fact that part of the produced argon atoms decay already in the tank before the chemical extraction. Assume that the chemical extraction was fast, i.g. the argon atoms were extracted imminently into to the proportional counter after the exposure time of one month.

4. **Proton decay, stability of matter**
Assume a half-life of 10^{34} years for protons. Estimate how many proton decays will happen in a 1 megaton Water-Cherenkov detector during its operation time of 30 years. Sir Arthur Eddington estimated in 1938 that the number of protons in the Universe is $1.57 * 10^{79}$. When would the last proton of the universe decay?