1. **Detection techniques II, simplified models of LSC and WC**

In this task, we study simple models of large-size LSC and WC in order to qualitatively understand their neutrino detection principle.

a) Assume a liquid scintillator tank of a cylindrical shape and radius \( r = 15 \) metres. Neutrinos can be detected by observing the following scattering reaction: \( \nu + e \rightarrow \nu + e \). In the scattering process, the neutrino energy is partly transferred into the electron.

The scattered electron excites and ionizes the scintillating medium and by its energy losses is stopped in the medium. Assume, for simplicity, that the light emission of the scintillator is monochromatic with \( \lambda \approx 280 \) nm and that the light yield is \( \epsilon_{SC} = 0.1 \). Also assume an attenuation length of \( \lambda \approx 20 \) m. Assume that one photomultiplier of the front cover diameter size of 8\(^\prime\) is attached to the wall of the tank.

Consider a neutrino scattering reaction yielding an electron of the energy of 0.25 MeV at the very centre of the tank. How many photons would arrive to the photomultiplier surface? What if the electron energy was 2.5 MeV? What about 25 MeV? Hint: the electron ranges are of the order of centimetres. You may simplify that the photons are generated in one point.

Let us define a photomultiplier efficiency \( \epsilon_{PMT} \) describing the fraction of photons converted into a measurable signal by the photomultiplier (i.e. \( \epsilon_{PMT} \) includes such technical features as the light collection efficiency, the quantum efficiency of photocathode, etc.). We may take \( \epsilon_{PMT} = 0.2 \). If one would place a number of photomultipliers side by side around the ring-shaped wall, how many photomultipliers would fire, i.e. record a photon?

b) Neutrino detection in a Water-Cherenkov detector may be based on the observation of the scattering reaction: \( \nu + e^- \rightarrow \nu + e^- \). In the scattering process, the incident neutrino energy is partly transferred into the electron.

Formulate the expression of the angle between the direction of the incident neutrino and the direction of the scattered electron in terms of the electron mass, electron kinetic energy and neutrino energy. Hint: the energy and momentum are conserved in the scattering process.

Take that the minimum observable electron energy is \( \approx 5 \) MeV and a maximum solar neutrino energy is \( \approx 20 \) MeV. Calculate the maximum value for the angle between the incident neutrino and the electron.

Assume a cylindrical detector of the radius of 17 metres, that photomultipliers cover 40\% of the surface of the detector wall, that the photomultiplier efficiency is \( \epsilon_{PMT} = 0.2 \) and that the light attenuation is \( \lambda = 100 \) metres. Assume an electron of the energy between 1 – 10 MeV causing Cherenkov radiation. Approximate the
range of electron by \( R \approx E_e/2 \) cm and, given that the number of Cherenkov-photons generated per electron path length is \( n \approx 300 \) photons/cm, estimate the number of fired photomultipliers as a function of electron energy.

c) Give examples about how one could improve these detector models.

2. \textbf{Radiochemical neutrino measurements II, Gallium Neutrino Observatory (GNO)}
Radiochemical neutrino experiments use isotopes with relatively high cross-sections for inverse beta decay (neutrino capture of a proton). After some exposure time, the decay of the created instable nuclei is measured to compute the number of neutrino events. Try to answer the following questions about the GNO experiment, which consisted of a 100-ton Gallium Chloride target containing 30.3 tons of Gallium:
(a) GNO experiment measured the reaction \( \nu_e + ^{71}\text{Ga} \rightarrow ^{71}\text{Ge} + e^- \).

What is the threshold energy for the reaction?
(b) What kind of background has to be considered?
(c) The neutrino capture rate is usually given independent from the target mass in solar neutrino units (SNU). 1 SNU is equal to one \( \nu_e \) capture per \( 10^{36} \) target atoms per second. The predicted rate for \( ^{71}\text{Ga} \) is 129 SNU. How many \( ^{71}\text{Ge} \) atoms are produced in GNO during a 28 day solar run?
(d) Give the time-dependent differential equation for the number of \( ^{71}\text{Ge} \) nuclei. Take into account the creation of \( ^{71}\text{Ge} \) with a constant production rate and the decay of \( ^{71}\text{Ge} \) with the lifetime \( \tau=16.5 \) d. What is the expected number of \( ^{71}\text{Ge} \) atoms in GNO at the end of a 28-day solar run?
(e) The actual measured rate by GNO is only 62.9±6.0 SNU. What could be possible explanations for this deviation? (Hint: The lecture slide 9.4).

For further reading about solar neutrino problem:

3. \textbf{Solar neutrino flux}
In the Sun’s core, energy is released through a series of thermonuclear fusion reactions, most importantly in the process called pp-chain. The most important branch of the chain is the ppI, where energy is released by the fusion of four protons into helium:
\[ 4 \ ^1\text{H} \rightarrow ^4\text{He} + 2e^+ + 2\nu_e \]
a) Calculate the energy released per \( ^4\text{He} \)-nucleus created.
b) The neutrinos produced have an average energy of 0.3 MeV. The remaining energy is radiated from the solar surface as electromagnetic radiation. Taken that
the solar irradiance at Earth’s average distance is 1.4 kW/m², how many \(^4\)He-nucleus are created in a second? How much matter is converted to energy each second?

c) Estimate the total flux of solar neutrinos at Earth. How do the neutrino oscillations affect this value? How does the flux change during a year?

d) Most of the neutrinos produced in the pp-chain come from the reaction

\[ ^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + \text{e}^+ + \nu_e \]

Calculate the maximum energy of the produced neutrino.

e) Neutrinos are also produced in ppII and ppIII branches through \(^7\)Be+e\(^-\) → \(^7\)Li+\(\nu_e\) and \(^8\)B→ \(^8\)Be+e\(^+\)+\(\nu_e\), in hep- and pep- reactions through \(^3\)He+p → \(^4\)He+e\(^+\)+\(\nu_e\), \(p + \text{e} + p \rightarrow ^2\text{H} + \nu_e\) and in the CNO-cycle through \(^{13}\)N→ \(^{13}\)C+e\(^+\)+\(\nu_e\), \(^{15}\)O→ \(^{15}\)N+e\(^+\)+\(\nu_e\) and \(^{17}\)F→ \(^{17}\)O+e\(^+\)+\(\nu_e\). What can you say about the energy of the neutrinos produced in these reactions?

Hints: a) take into account positron annihilation \(\text{e}^+ + \text{e}^- \rightarrow \gamma + \gamma\).

4. Two messengers from the core of the Sun, the light, the neutrino

The energy released from the core is transported to the outer layers of the Sun by radiation.

a) Estimate the time required for a photon (energy) to propagate from the centre of the Sun to its surface.

Hints:
- use random walk approximation to describe the travel of the photon in the Sun. Assume that the photon is absorbed and re-emitted in a random direction after a step of the length of \(\lambda\).
- you may use an average value for the mean free path of a photon: \(\lambda = 10^{-3}\) metres.

For further reading: http://sunearthday.nasa.gov/2007/locations/ttt_sunlight.php

b) Estimate the time for a neutrino to travel from the center of the Sun to its surface.

c) Are there any other messengers from the Sun?