1. **Neutrino mixing**

a) The MNSP matrix can be factorized as

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} = U_{12} \times U_{23} \times U_{13}$$

with

$$U_{12} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \text{and}\, U_{23} = \begin{pmatrix} c_{13} & 0 & s_{13}e^{i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$, and the phase factor δ contains the possible CP-violation. Calculate matrix U.

b) Calculate the effective neutrino mass $\langle m_\nu \rangle = \sum_i U_{ei}^2 m_i$.

2. **Dark Matter I, Galactic Halo:**

The rotation curve of galaxy NGC6503 [lecture notes 13.3] is quite flat at large distances 3 to 23 kpc. Calculate the density of the galactic halo $\rho(r)$ assuming that the rotation velocity v_c is constant. Draw the density of the galactic halo when the radius r goes from 3 to 23 kpc.

Hint: $GM(< r)/r = v_c^2$ and $M(< r) = \int_0^r 4\pi r^2 \rho(r) dr$.

3. **Dark Matter II, WIMP:** WIMP's can be detected via elastic scattering of nuclei of the detector material. Assume a WIMP with m_{WIMP}=10 GeV/c^2 hitting a Ge nucleus with a relative velocity of v=100 km/s. What is the transferred energy in a central collision? How does this compare to the energy transfer of a WIMP collision with an electron of the Ge atom?