Generalised Linear Mixed Models

Jarrod Hadfield

University of Edinburgh

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Generalised Linear Mixed Models

- ANCOVA
- Bradley-Terry models
- MANCOVA
- Meta-analysis
- Multi-membership models
- Pedigree analysis: animal models
- Phylogenetic analysis: comparative approach
- Random Regression
- Rasch Models
- Regression
- Ridge Regression
- Splines
- Survival-analysis
- Threshold models
- Time-series
- Varying coefficient models

Outline

- What is a linear model?
- What is a random effect?
- MCMCglmm
- Non-Gaussian data
- Structured random effects

Linear Model

```
> data(Traffic, package="MASS")
```

A Swedish Experiment: On some days make everyone drive to the speed limit on others let everyone drive as fast as they want. Count how many citizens are killed.

Linear Model

```
> data(Traffic, package="MASS")
```

A Swedish Experiment: On some days make everyone drive to the speed limit on others let everyone drive as fast as they want. Count how many citizens are killed.

```
> Traffic[c(1,2,184),]
```

```
    year day limit  y
1   1961   1    no  9
2   1961   2    no 11
184 1962  92   yes  9
```

Linear Model

- Model Syntax

```
y ~ limit + year + day
```

Linear Model

- Model Syntax

```
y ~ limit + year + day
```

- Set of Simultaneous Equations

$$
\begin{aligned}
E[y[1]] &= 1\beta_1 + (\texttt{limit[1]=="yes"})\beta_2 + (\texttt{year[1]=="1962"})\beta_3 + \texttt{day[1]}\beta_4 \\
E[y[2]] &= 1\beta_1 + (\texttt{limit[2]=="yes"})\beta_2 + (\texttt{year[2]=="1962"})\beta_3 + \texttt{day[2]}\beta_4 \\
\vdots &= \vdots \\
E[y[184]] &= 1\beta_1 + (\texttt{limit[184]=="yes"})\beta_2 + (\texttt{year[184]=="1962"})\beta_3 + \texttt{day[184]}\beta_4
\end{aligned}
$$

Linear Model

- Model Syntax

```
y ~ limit + year + day
```

- Set of Simultaneous Equations

$$
\begin{aligned}
E[y\texttt{[1]}] &= 1\beta_1 + (\texttt{limit[1]=="yes"})\beta_2 + (\texttt{year[1]=="1962"})\beta_3 + \texttt{day[1]}\beta_4 \\
E[y\texttt{[2]}] &= 1\beta_1 + (\texttt{limit[2]=="yes"})\beta_2 + (\texttt{year[2]=="1962"})\beta_3 + \texttt{day[2]}\beta_4 \\
\vdots &= \vdots \\
E[y\texttt{[184]}] &= 1\beta_1 + (\texttt{limit[184]=="yes"})\beta_2 + (\texttt{year[184]=="1962"})\beta_3 + \texttt{day[184]}\beta_4
\end{aligned}
$$

- Compact representation: design matrix and parameter vector

$$E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$$

Linear Model

- Model Syntax

```
y ~ limit + year + day
```

- Set of Simultaneous Equations

$$
\begin{aligned}
E[\texttt{y[1]}] &= 1\beta_1 + (\texttt{limit[1]=="yes"})\beta_2 + (\texttt{year[1]=="1962"})\beta_3 + \texttt{day[1]}\,\beta_4 \\
E[\texttt{y[2]}] &= 1\beta_1 + (\texttt{limit[2]=="yes"})\beta_2 + (\texttt{year[2]=="1962"})\beta_3 + \texttt{day[2]}\,\beta_4 \\
\vdots &= \vdots \\
E[\texttt{y[184]}] &= 1\beta_1 + (\texttt{limit[184]=="yes"})\beta_2 + (\texttt{year[184]=="1962"})\beta_3 + \texttt{day[184]}\,\beta_4
\end{aligned}
$$

- Compact representation: design matrix and parameter vector

$$E[\mathbf{y}] = \mathbf{X}\boldsymbol{\beta}$$

```
> X<-model.matrix(y~limit+year+day, data=Traffic)
> X[c(1,2,184),]
    (Intercept) limityes year1962 day
1             1        0        0   1
2             1        0        0   2
184           1        1        1  92
```

Linear Model

$$E[\mathbf{y}] = \mathbf{X}\beta$$

Linear Model

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- The full model

$$\mathbf{y} \sim N(\mathbf{X}\beta, \sigma^2_e\mathbf{I})$$

Linear Model

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- The full model

$$\mathbf{y} \sim N(\mathbf{X}\beta, \sigma^2_e\mathbf{I})$$

- Error structure

$$\sigma^2_e\mathbf{I} = \sigma^2_e \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \sigma^2_e & 0 & \dots & 0 \\ 0 & \sigma^2_e & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \sigma^2_e \end{bmatrix}$$

Linear Model

```
> m1<-MCMCglmm(y ~ limit + year + day, data=Traffic)
```

Linear Model

```
> m1<-MCMCglmm(y ~ limit + year + day, data=Traffic)

> summary(m1)

 Iterations = 3001:12991
 Thinning interval  = 10
 Sample size  = 1000

 DIC: 1314.586

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units     72.77     58.4    89.54     1000

 Location effects: y ~ limit + year + day

            post.mean  l-95% CI  u-95% CI eff.samp  pMCMC
(Intercept) 21.142478 18.285644 23.980493    802.2 <0.001 ***
limityes    -3.673001 -6.373628 -1.198028   1112.8  0.008 **
year1962    -1.341449 -4.113877  1.010848   1000.0  0.288
day          0.053102  0.005917  0.096801    854.9  0.024 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Linear Mixed Model

- Random Effects:

$$
\begin{aligned}
E[y\,[1]] &= \mathbf{X}[1,]\beta \\
E[y\,[2]] &= \mathbf{X}[2,]\beta \\
E[y\,[184] &= \mathbf{X}[184,]\beta
\end{aligned}
$$

Linear Mixed Model

- Random Effects:

$$
\begin{aligned}
E[y[1]] &= \mathbf{X}[1,]\beta + (\texttt{day[1]=="1"})u_1 + (\texttt{day[1]=="2"})u_2 \ldots (\texttt{day[1]=="92"})u_{92} \\
E[y[2]] &= \mathbf{X}[2,]\beta + (\texttt{day[2]=="1"})u_1 + (\texttt{day[2]=="2"})u_2 \ldots (\texttt{day[2]=="92"})u_{92} \\
E[y[184] &= \mathbf{X}[184,]\beta + (\texttt{day[184]=="1"})u_1 + (\texttt{day[184]=="2"})u_2 \ldots (\texttt{day[184]=="92"})u_{92}
\end{aligned}
$$

Linear Mixed Model

- Random Effects:

$$
\begin{aligned}
E[y\texttt{[1]}] &= \mathbf{X}[1,]\beta + \texttt{(day[1]=="1")}u_1 + \texttt{(day[1]=="2")}u_2 \ldots \texttt{(day[1]=="92")}u_{92} \\
E[y\texttt{[2]}] &= \mathbf{X}[2,]\beta + \texttt{(day[2]=="1")}u_1 + \texttt{(day[2]=="2")}u_2 \ldots \texttt{(day[2]=="92")}u_{92} \\
E[y\texttt{[184]} &= \mathbf{X}[184,]\beta + \texttt{(day[184]=="1")}u_1 + \texttt{(day[184]=="2")}u_2 \ldots \texttt{(day[184]=="92")}u_{92}
\end{aligned}
$$

- Compact representation: design matrix and parameter vector

$$
\begin{aligned}
E[\mathbf{y}] &= \mathbf{X}\beta + \mathbf{Z}\mathbf{u} \\
&= \mathbf{W}\theta
\end{aligned}
$$

Linear Mixed Model

- Random Effects:

$$
\begin{aligned}
E[y\texttt{[1]}] &= \mathbf{X}[1,]\beta + \texttt{(day[1]=="1")}u_1 + \texttt{(day[1]=="2")}u_2 \ldots \texttt{(day[1]=="92")}u_{92}\\
E[y\texttt{[2]}] &= \mathbf{X}[2,]\beta + \texttt{(day[2]=="1")}u_1 + \texttt{(day[2]=="2")}u_2 \ldots \texttt{(day[2]=="92")}u_{92}\\
E[y\texttt{[184]} &= \mathbf{X}[184,]\beta + \texttt{(day[184]=="1")}u_1 + \texttt{(day[184]=="2")}u_2 \ldots \texttt{(day[184]=="92")}u_{92}
\end{aligned}
$$

- Compact representation: design matrix and parameter vector

$$
\begin{aligned}
E[\mathbf{y}] = & \quad \mathbf{X}\beta + \mathbf{Z}\mathbf{u}\\
= & \quad \mathbf{W}\boldsymbol{\theta}
\end{aligned}
$$

$$
\boldsymbol{\theta} = \left[\begin{array}{c} \beta \\ \mathbf{u} \end{array} \right]
$$

Linear Mixed Model

- Random Effects:

$$\begin{aligned} E[y\texttt{[1]}] &= \mathbf{X}[1,]\beta + \texttt{(day[1]=="1")}u_1 + \texttt{(day[1]=="2")}u_2 \ldots \texttt{(day[1]=="92")}u_{92} \\ E[y\texttt{[2]}] &= \mathbf{X}[2,]\beta + \texttt{(day[2]=="1")}u_1 + \texttt{(day[2]=="2")}u_2 \ldots \texttt{(day[2]=="92")}u_{92} \\ E[y\texttt{[184]} &= \mathbf{X}[184,]\beta + \texttt{(day[184]=="1")}u_1 + \texttt{(day[184]=="2")}u_2 \ldots \texttt{(day[184]=="92")}u_{92} \end{aligned}$$

- Compact representation: design matrix and parameter vector

$$\begin{aligned} E[\mathbf{y}] &= \mathbf{X}\beta + \mathbf{Z}\mathbf{u} \\ &= \mathbf{W}\boldsymbol{\theta} \end{aligned}$$

$$\boldsymbol{\theta} = \begin{bmatrix} \beta \\ \mathbf{u} \end{bmatrix}$$

$$\mathbf{W} = [\mathbf{X}, \mathbf{Z}]$$

Linear Mixed Model

- Random Effects:

$$
\begin{aligned}
E[y\texttt{[1]}] &= \mathbf{X}[1,]\beta + (\texttt{day[1]=="1"})u_1 + (\texttt{day[1]=="2"})u_2 \ldots (\texttt{day[1]=="92"})u_{92} \\
E[y\texttt{[2]}] &= \mathbf{X}[2,]\beta + (\texttt{day[2]=="1"})u_1 + (\texttt{day[2]=="2"})u_2 \ldots (\texttt{day[2]=="92"})u_{92} \\
E[y\texttt{[184]} &= \mathbf{X}[184,]\beta + (\texttt{day[184]=="1"})u_1 + (\texttt{day[184]=="2"})u_2 \ldots (\texttt{day[184]=="92"})u_{92}
\end{aligned}
$$

- Compact representation: design matrix and parameter vector

$$
\begin{aligned}
E[\mathbf{y}] &= \mathbf{X}\beta + \mathbf{Z}\mathbf{u} \\
&= \mathbf{W}\boldsymbol{\theta}
\end{aligned}
$$

$$
\boldsymbol{\theta} = \left[\begin{array}{c} \beta \\ \mathbf{u} \end{array} \right]
$$

$$
\mathbf{W} = [\mathbf{X}, \mathbf{Z}]
$$

```
> Z<-model.matrix(~as.factor(day)-1, data=Traffic)
> W<-cbind(X,Z)
```

Linear Mixed Model

- Fixed Effects

$$\boldsymbol{\beta} \sim N(\mathbf{0}, \sigma^2_{\beta}\mathbf{I})$$

σ^2_{β} is not estimated, and is usually assumed to be large (or often ∞ in non-Bayesian models)

Linear Mixed Model

- Fixed Effects

$$\boldsymbol{\beta} \sim N(\mathbf{0}, \sigma^2_{\beta}\mathbf{I})$$

σ^2_{β} is not estimated, and is usually assumed to be large (or often ∞ in non-Bayesian models)

- Random Effects

$$\mathbf{u} \sim N(\mathbf{0}, \sigma^2_{u}\mathbf{I})$$

σ^2_{u} is estimated.

Linear Mixed Model

```
> m2<-MCMCglmm(y ~ limit + year + day, random=~day, data=Traffic)
```

Linear Mixed Model

```
> m2<-MCMCglmm(y ~ limit + year + day, random=~day, data=Traffic)

> summary(m2)

 Iterations = 3001:12991
 Thinning interval  = 10
 Sample size  = 1000

 DIC: 1196.909

 G-structure:  ~day

    post.mean l-95% CI u-95% CI eff.samp
day     47.49    29.21    65.92     1000

 R-structure:  ~units

      post.mean l-95% CI u-95% CI eff.samp
units      26.2    19.31    34.18     1000

 Location effects: y ~ limit + year + day

            post.mean l-95% CI u-95% CI eff.samp  pMCMC
(Intercept)  21.55419 18.48638 24.75515     1000 <0.001 ***
limityes     -5.41994 -7.34187 -3.61084     1000 <0.001 ***
year1962     -0.83743 -2.51252  0.57858     1000  0.288
day           0.05186 -0.01099  0.10184     1000  0.078 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Linear Mixed Model: Credible Intervals

> *plot(m2$VCV)*

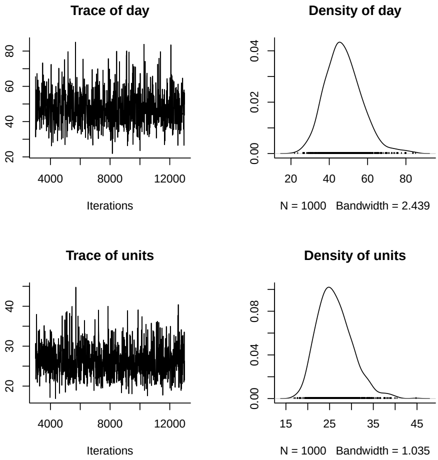

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for the variance components.

Linear Mixed Model: Credible Intervals

```
> plot(cbind(m2$VCV), type="l")
```

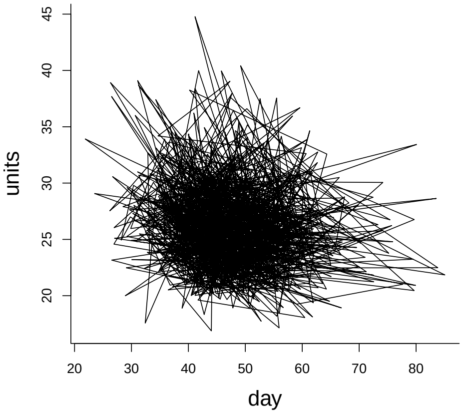

Figure: MCMC trace through the joint posterior distribution for the two variance components.

Linear Mixed Model: Credible Intervals

```
> r2<-m2$VCV[,"day"]/(m2$VCV[,"day"]+m2$VCV[,"units"])
```

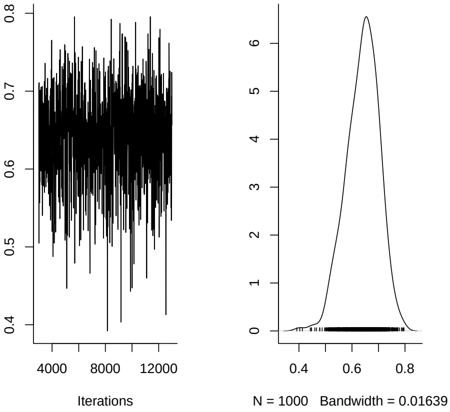

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for the proportion of variance explained by day.

Linear Model Diagnostics

```
> hist(Traffic$y-predict(m2))
```

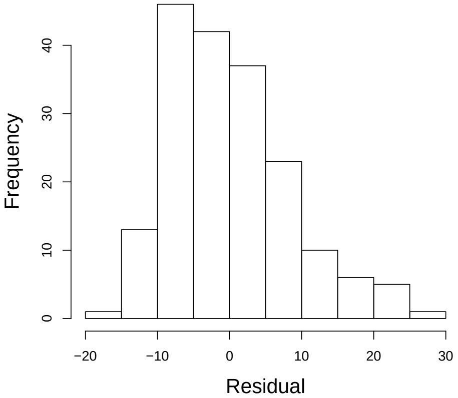

Figure: Histogram of residuals from model `m1` which assumes they followed a Normal distribution.

Generalised Linear Model

- Link function $g()$: log

$$\log(E[\mathbf{y}]) = \mathbf{X}\beta$$

Generalised Linear Model

- Link function $g()$: log

$$
\begin{aligned}
\log(E[\mathbf{y}]) &= \mathbf{X}\boldsymbol{\beta} \\
E[\mathbf{y}] &= \log^{-1}(\mathbf{X}\boldsymbol{\beta}) \\
E[\mathbf{y}] &= \exp(\mathbf{X}\boldsymbol{\beta})
\end{aligned}
$$

Generalised Linear Model

- Link function $g()$: log

$$\log(E[\mathbf{y}]) = \mathbf{X}\boldsymbol{\beta}$$
$$E[\mathbf{y}] = \log^{-1}(\mathbf{X}\boldsymbol{\beta})$$
$$E[\mathbf{y}] = \exp(\mathbf{X}\boldsymbol{\beta})$$

- Distribution: Poisson

$$\mathbf{y} \sim Pois(\lambda = \exp(\mathbf{X}\boldsymbol{\beta}))$$

Generalised Linear Mixed Model: MCMCglmm

- A 'latent' variable ℓ where $g^{-1}(\ell)$ is the distribution parameter:

Generalised Linear Mixed Model: MCMCglmm

- A 'latent' variable $\boldsymbol{\ell}$ where $g^{-1}(\boldsymbol{\ell})$ is the distribution parameter:

$$\mathbf{y} \sim \mathit{Pois}(\lambda = \exp(\boldsymbol{\ell}))$$

Generalised Linear Mixed Model: MCMCglmm

- A 'latent' variable $\boldsymbol{\ell}$ where $g^{-1}(\boldsymbol{\ell})$ is the distribution parameter:

$$\mathbf{y} \sim \mathit{Pois}(\lambda = \exp(\boldsymbol{\ell}))$$

- then apply a standard linear model for the 'latent' variables:

$$\boldsymbol{\ell} \sim N(\mathbf{W}\boldsymbol{\theta}, \mathbf{I}\sigma^2_e)$$

Generalised Linear Mixed Model: MCMCglmm

- A 'latent' variable $\boldsymbol{\ell}$ where $g^{-1}(\boldsymbol{\ell})$ is the distribution parameter:

$$\mathbf{y} \sim Pois(\lambda = \exp(\boldsymbol{\ell}))$$

- then apply a standard linear model for the 'latent' variables:

$$\boldsymbol{\ell} \sim N(\mathbf{W}\boldsymbol{\theta}, \mathbf{I}\sigma^2_e)$$

Standard Poisson glm assumes $\sigma^2_e = 0$.

Generalised Linear Model: Poisson

```
> m3<-MCMCglmm(y ~ limit + year + day, data=Traffic, family="poisson")
```

Generalised Linear Model: Poisson

```
> m3<-MCMCglmm(y ~ limit + year + day, data=Traffic, family="poisson")

> summary(m3)

 Iterations = 3001:12991
 Thinning interval  = 10
 Sample size  = 1000

 DIC: 1196.587

 R-structure:  ~units

     post.mean l-95% CI u-95% CI eff.samp
units    0.1005  0.07111   0.1326    766.2

 Location effects: y ~ limit + year + day

             post.mean   l-95% CI   u-95% CI eff.samp  pMCMC
(Intercept)  2.9930471  2.8507838  3.1213949   1000.0 <0.001 ***
limityes    -0.1722347 -0.2965726 -0.0607209    893.8  0.010 **
year1962    -0.0656572 -0.1932587  0.0408528   1000.0  0.288
day          0.0025720  0.0004386  0.0045855   1034.7  0.016 *
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Generalised Linear Model: Poisson

```
> prior<-list(R=list(V=0.01, fix=1))
> m4<-MCMCglmm(y ~ limit + year + day, data=Traffic, family="poisson", prior=prior)
```

Generalised Linear Model: Poisson

```
> prior<-list(R=list(V=0.01, fix=1))
> m4<-MCMCglmm(y ~ limit + year + day, data=Traffic, family="poisson", prior=prior)

> summary(m4)

 Iterations = 3001:12991
 Thinning interval  = 10
 Sample size  = 1000

 DIC: 1339.032

 R-structure:  ~units

     post.mean l-95% CI u-95% CI eff.samp
units      0.01     0.01     0.01        0

 Location effects: y ~ limit + year + day

            post.mean  l-95% CI  u-95% CI eff.samp  pMCMC
(Intercept)  3.038029  2.956563  3.131036    407.8 <0.001 ***
limityes    -0.177145 -0.263460 -0.105055    322.3 <0.001 ***
year1962    -0.059932 -0.133893  0.010275    336.6  0.092 .
day          0.002479  0.001254  0.003844    390.3 <0.001 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Generalised Linear Mixed Model: Poisson

```
> m5<-MCMCglmm(y ~ limit + year + day, random=~day, data=Traffic, family="poisson")
```

Generalised Linear Mixed Model: Poisson

```
> m5<-MCMCglmm(y ~ limit + year + day, random=~day, data=Traffic, family="poisson")
> summary(m5)

 Iterations = 3001:12991
 Thinning interval  = 10
 Sample size  = 1000

 DIC: 1137.102

 G-structure:  ~day

    post.mean l-95% CI u-95% CI eff.samp
day   0.08492  0.05985   0.1116    123.6

 R-structure:  ~units

      post.mean  l-95% CI u-95% CI eff.samp
units 0.0001814 3.554e-07  0.00105    14.63

 Location effects: y ~ limit + year + day

            post.mean  l-95% CI  u-95% CI eff.samp  pMCMC
(Intercept)  3.051203  2.910127  3.179769   64.792 <0.001 ***
limityes    -0.276365 -0.327630 -0.241085    8.493 <0.001 ***
year1962    -0.044387 -0.066061 -0.018568   21.757   0.01 **
day          0.002207 -0.000182  0.004692   67.455   0.08 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

Generalised Linear Mixed Model: Poisson

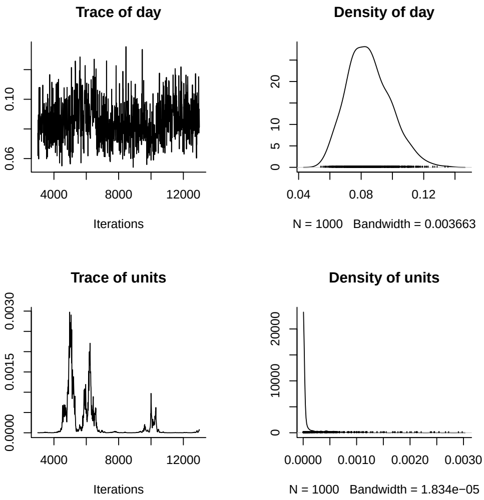

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) with flat (improper) priors on the variance components.

Generalised Linear Model

```
> prior<-list(R=list(V=1, nu=0.002), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
```

Generalised Linear Model

```
> prior<-list(R=list(V=1, nu=0.002), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m5.b<-MCMCglmm(y ~ limit + year + day, random=~day, data=Traffic, family="poisson",
+   prior=prior, nitt=13000*10, thin=10*10, burnin=3000*10)
> plot(m5.b$VCV)
```

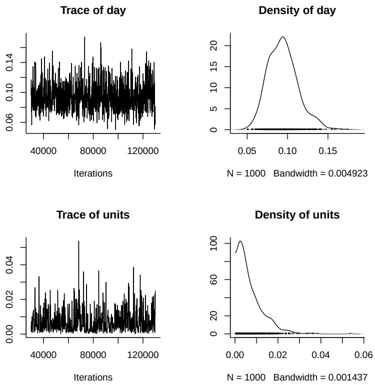

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) with proper priors on the variance components.

Distributions

- Binomial
- Multinomial
- Gaussian
- Poisson
- Ordinal
- Exponential
- Geometric
- Threshold
- Zero-inflated Poisson
- Zero-altered Poisson
- Hurdle Poisson
- Zero-inflated Binomial
- Censored Gaussian
- Censored Poisson
- Censored Exponential

Generalised Linear Mixed Model: Binary

- A 'latent' variable ℓ where $g^{-1}(\ell)$ is the distribution parameter:

Generalised Linear Mixed Model: Binary

- A 'latent' variable $\boldsymbol{\ell}$ where $g^{-1}(\boldsymbol{\ell})$ is the distribution parameter:

$$\mathbf{y} \sim Binom(Pr = \mathrm{probit}^{-1}(\boldsymbol{\ell}))$$

- then apply a standard linear model for the 'latent' variables:

$$\boldsymbol{\ell} \sim N(\mathbf{W}\boldsymbol{\theta}, \mathbf{I}\sigma^2_e)$$

Generalised Linear Mixed Model: Binary

```
> Traffic$y2<-as.numeric(Traffic$y>20)
> m6<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+   family="ordinal", slice=TRUE)
```

Generalised Linear Mixed Model: Binary

```
> Traffic$y2<-as.numeric(Traffic$y>20)
> m6<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+   family="ordinal", slice=TRUE)
> plot(m6$VCV)
```

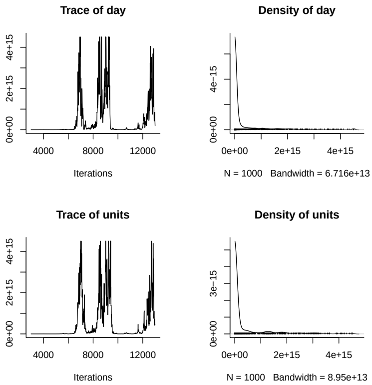

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for variance components.

Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=1, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m7<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="ordinal", slice=TRUE, prior=prior)
```

Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=1, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m7<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="ordinal", slice=TRUE, prior=prior)
> plot(m7$VCV)
```

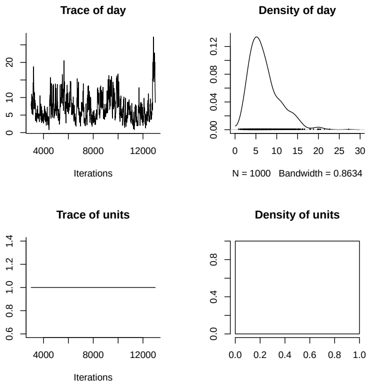

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for variance components.

Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=0.5, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m8<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="ordinal", slice=TRUE, prior=prior)
```

Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=0.5, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m8<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="ordinal", slice=TRUE, prior=prior)
> plot(mcmc.list(m7$VCV, m8$VCV))
```

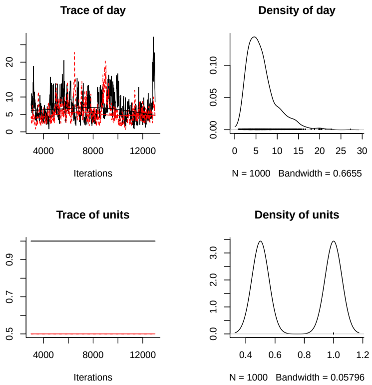

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for variance components with $\sigma^2_e = 1$ (black) and $\sigma^2_e = 0.5$ (red)

```
> plot(mcmc.list(m7$Sol[,"limityes"], m8$Sol[,"limityes"]))
```

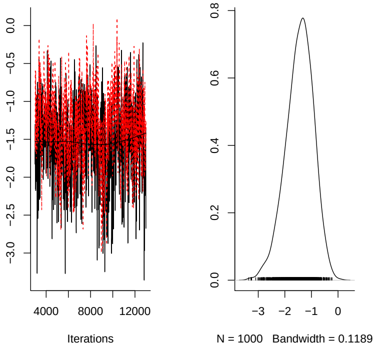

Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for the effect of a speed limit with $\sigma^2_e = 1$ (black) and $\sigma^2_e = 0.5$ (red)

Generalised Linear Mixed Model: Binary

```
> res.7<-m7$Sol[,"limityes"]/sqrt(m7$VCV[,"units"]+1)
> res.8<-m8$Sol[,"limityes"]/sqrt(m8$VCV[,"units"]+1)
```

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```
> res.7<-m7$Sol[,"limityes"]/sqrt(m7$VCV[,"units"]+1)
> res.8<-m8$Sol[,"limityes"]/sqrt(m8$VCV[,"units"]+1)
> plot(mcmc.list(res.7, res.8))
```

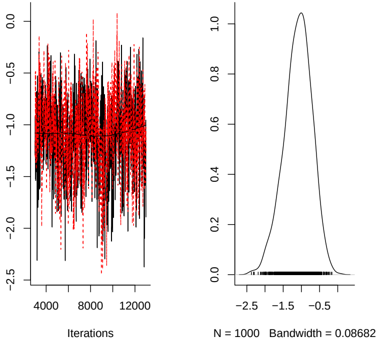
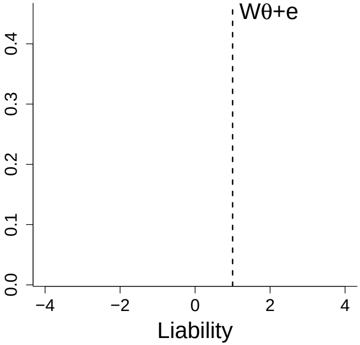


Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for the *scaled* effect of a speed limit with $\sigma^2_e = 1$ (black) and $\sigma^2_e = 0.5$ (red)

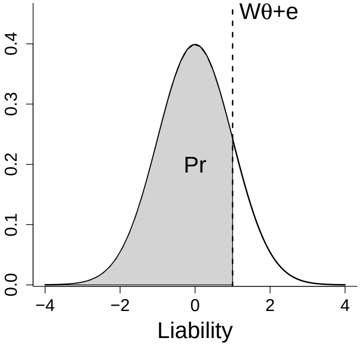
Generalised Linear Mixed Model: Binary

$$Pr = \text{probit}^{-1}(\ell)$$



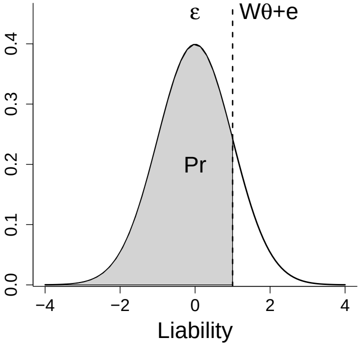
Generalised Linear Mixed Model: Binary

$$Pr = \mathrm{probit}^{-1}(\boldsymbol{\ell})$$

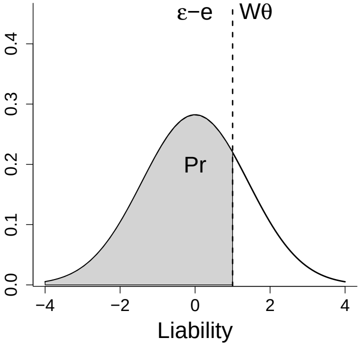


Generalised Linear Mixed Model: Binary

$$Pr = \text{probit}^{-1}(\boldsymbol{\ell})$$

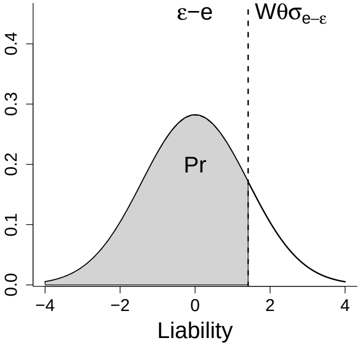


$$Pr = \text{probit}^{-1}(\boldsymbol{\ell})$$



Generalised Linear Mixed Model: Binary

$$Pr = \text{probit}^{-1}(\ell)$$



Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=1, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m9<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="threshold", prior=prior)
```

Generalised Linear Mixed Model: Binary

```
> prior<-list(R=list(V=1, fix=1), G=list(G1=list(V=1, nu=1, alpha.mu=0, alpha.V=1000)))
> m9<-MCMCglmm(y2 ~ limit + year + day, random=~day, data=Traffic,
+     family="threshold", prior=prior)
> plot(mcmc.list(res.7, res.8, m9$Sol[,"limityes"]))
```

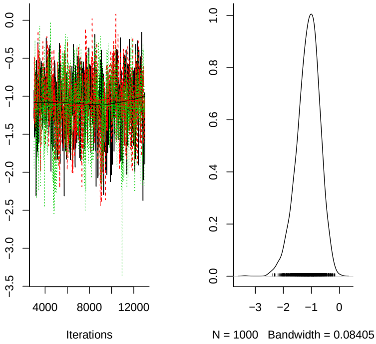


Figure: Time-series of MCMC output (left) and smoothed posterior distribution (right) for the *scaled* effect of a speed limit with $\sigma^2_e = 1$ (black), $\sigma^2_e = 0.5$ (red) and $\sigma^2_e = 0$ (green)